1 We know that the point $P(x,y)$ satisfies,

$$QP = 4$$
$$\sqrt{(x-1)^2+(y-(-2))^2} = 4$$
$$(x-1)^2+(y+2)^2 = 4^2.$$

This is a circle with centre $(1,-2)$ and radius 4.

2 We know that the point $P(x,y)$ satisfies,

$$QP = 5$$
$$\sqrt{(x-(-4))^2+(y-3)^2} = 5$$
$$(x+4)^2+(y-3)^2 = 5^2.$$

This is a circle with centre $(-4,3)$ and radius 5.

3 a We know that the point $P(x,y)$ satisfies,

$$QP = RP$$
$$\sqrt{(x-(-1))^2+(y-(-1))^2} = \sqrt{(x-1)^2+(y-1)^2}$$
$$(x+1)^2+(y+1)^2 = (x-1)^2+(y-1)^2$$
$$x^2+2x+1+y^2+2y+1 = x^2-2x+1+y^2-2y+1$$
$$2x+2y+ = -2x-2y$$
$$y = -x$$

b The above line has gradient -1. The straight line through $Q(-1,-1)$ and $R(1,1)$ has equation

$$y = x$$

and thus has gradient 1. Because the product of the two gradients is -1, the two lines are perpendicular. Lastly, the midpoint of points Q and R is $(0,0)$. This point is on the line $y=-x$ since if $x=0$ then $y=0$.

4 a We know that the point $P(x,y)$ satisfies,

$$QP = RP$$
$$\sqrt{(x-0)^2+(y-2)^2} = \sqrt{(x-1)^2+y^2}$$
$$x^2+(y-2)^2 = (x-1)^2+y^2$$
$$x^2+y^2-4y+4 = x^2-2x+1+y^2$$
$$-4y+4 = -2x+1$$
$$y = \frac{x}{2}+\frac{3}{4}$$

b The above line has gradient $1/2$. The straight line through $Q(0,2)$ and $R(1,0)$ has gradient

$$m = \frac{0-2}{1-0} = -2$$

and equation

$$y = -2x+2.$$

Because the product of the two gradients is -1, the two lines are perpendicular. Lastly, the midpoint of points Q and R is $(1/2,1)$. This point is on the line

$$y = \frac{x}{2}+\frac{3}{4}$$

since if $x=1/2$ then

$$y = 1/4+3/4 = 1.$$

5 Since $P(x,y)$ is equidistant from points $Q(0,1)$ and $R(2,3)$ we know that

$$QP = RP$$
$$\sqrt{x^2+(y-1)^2} = \sqrt{(x-2)^2+(y-3)^2}$$

$$
\begin{aligned}
x^2 + (y-1)^2 &= (x-2)^2 + (y-3)^2 \\
x^2 + y^2 - 2y + 1 &= x^2 - 4x + 4 + y^2 - 6y + 9 \\
-2y + 1 &= -4x - 6y + 13 \\
4y + 4x &= 12 \\
y = -x + 3 & \qquad (1)
\end{aligned}
$$

We also know that $P(x, y)$ is 3 units away from $S(3, 3)$. Therefore $P(x, y)$ must lie on the circle whose equation is

$$(x-3)^2 + (y-3)^2 = 3^2. \qquad (2)$$

Substituting equation (1) into equation (2) gives

$$
\begin{aligned}
(x-3)^2 + (-x+3-3)^2 &= 9 \\
(x-3)^2 + x^2 &= 9 \\
x^2 - 6x + 9 + x^2 &= 9 \\
2x^2 - 6x &= 0 \\
2x(x-3) &= 0
\end{aligned}
$$

Therefore, either $x = 0$ or $x = 3$. Substituting $x = 0$ into (1) gives $y = 3$. Substituting $x = 3$ into (1) gives $y = 0$. Therefore, there are two answers: coordinates $(0, 3)$ and $(3, 0)$.

6 Since $P(x, y)$ is equidistant from points $Q(0, 1)$ and $R(2, 0)$ we know that

$$
\begin{aligned}
QP &= RP \\
\sqrt{x^2 + (y-1)^2} &= \sqrt{(x-2)^2 + y^2} \\
x^2 + y^2 - 2y + 1 &= x^2 - 4x + 4 + y^2 \\
-2y + 1 &= -4x + 4 \\
-2y &= -4x + 3 \\
4x - 2y = 3. & \qquad (1)
\end{aligned}
$$

Since $P(x, y)$ is equidistant from points $S(-1, 0)$ and $T(0, 2)$ we know that

$$
\begin{aligned}
SP &= TP \\
\sqrt{(x+1)^2 + y^2} &= \sqrt{x^2 + (y-2)^2} \\
x^2 + 2x + 1 + y^2 &= x^2 + y^2 - 4y + 4 \\
2x + 1 &= -4y + 4 \\
-4y &= 2x - 3 \\
2x + 4y = 3. & \qquad (2)
\end{aligned}
$$

Solving equations (1) and (2) simultaneously gives $x = \dfrac{9}{10}$ and $y = \dfrac{3}{10}$.

7 Since the treasure is 10 metres from a tree stump located at coordinates $T(0, 0)$, it lies on the circle whose equation is

$$x^2 + y^2 = 10^2. \qquad (1)$$

Since the treasure is 2 metres from a rock at coordinates $R(6, 10)$, it lies on the circle whose equation is

$$(x-6)^2 + (y-10)^2 = 2^2 \qquad (2)$$

Solving equations (1) and (2) simultaneously or by using your calculator gives two possible coordinates: either $(6, 8)$ or $\left(\dfrac{72}{17}, \dfrac{154}{17}\right)$.

8 a Since $P(x, y)$ is equidistant from points $R(4, 5)$ and $S(6, 1)$ we know that

$$
\begin{aligned}
RP &= SP \\
\sqrt{(x-4)^2 + (y-5)^2} &= \sqrt{(x-6)^2 + (y-1)^2} \\
x^2 - 8x + 16 + y^2 - 10y + 25 &= x^2 - 12x + 36 + y^2 - 2y + 1 \\
-8x + 16 - 10y + 25 &= -12x + 36 - 2y + 1 \\
8y - 4x &= 4 \\
2y - x = 1 & \qquad (1)
\end{aligned}
$$

b Since $P(x, y)$ is equidistant from points $S(6,1)$ and $T(1,-4)$ we know that

$$\begin{aligned} SP &= TP \\ \sqrt{(x-6)^2+(y-1)^2} &= \sqrt{(x-1)^2+(y+4)^2} \\ x^2-12x+36+y^2-2y+1 &= x^2-2x+1+y^2+8y+16 \\ -12x+36-2y+1 &= -2x+1+8y+16 \\ x+y &= 2 \qquad (1) \end{aligned}$$

c Solving equations (1) and (2) simultaneously gives $x=1$ and $y=1$ so that the point required is $P(1,1)$.

d The centre of the circle is $P(1,1)$ and its radius will be the distance from $P(1,1)$ to $R(4,5)$. This is

$$r = PR = \sqrt{(4-1)^2+(5-1)^2} = 5.$$

Therefore the equation of the circle must be

$$(x-1)^2+(y-1)^2 = 5^2.$$

9 Let the point be $P(x, y)$. The gradient of AB is

$$\frac{5-1}{2-0} = 2.$$

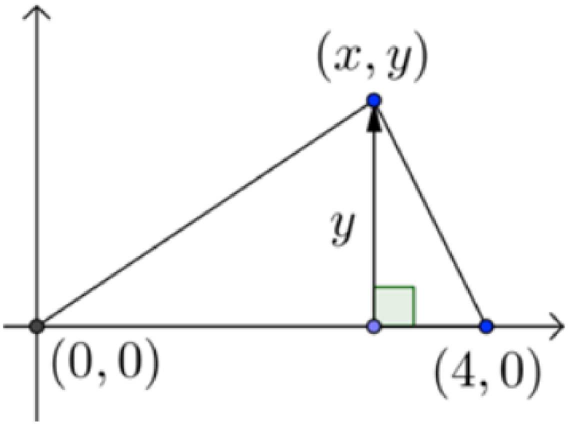
The gradient of BP is

$$\frac{y-5}{x-2}.$$

Equating the two gradients gives,

$$\begin{aligned} \frac{y-5}{x-2} &= 2 \\ y-5 &= 2(x-2) \\ y &= 2x+1. \end{aligned}$$

10 The triangle is shown below.



The base of the triangle has length 4 and its height is y. Therefore,

$$\begin{aligned} A &= \frac{bh}{2} \\ 12 &= \frac{4y}{2} \\ y &= 6. \end{aligned}$$

11a Let the point be $P(x, y)$. Then as the distance from P to the origin is equal to the sum of its x and y coordinates,

$$\begin{aligned} \sqrt{x^2+y^2} &= x+y \\ x^2+y^2 &= (x+y)^2 \\ x^2+y^2 &= x^2+2xy+y^2 \\ 2xy &= 0 \end{aligned}$$

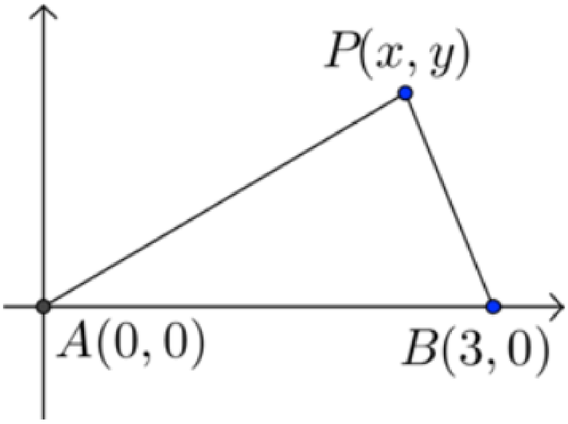
Therefore either $x=0$ or $y=0$. This is just both coordinate axes.

b Let the point be $P(x, y)$. Then as the distance from P to the origin is equal to the square of the sum of its x and y coordinates,

$$x^2 + y^2 = x + y$$
$$x^2 - x + y^2 - y = 0$$
$$\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \left(y^2 - y + \frac{1}{4}\right) - \frac{1}{4} = 0$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$

This is a circle with centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ with radius $\frac{1}{\sqrt{2}}$.

12 Consider point $P(x, y)$. The triangle is shown below.



We have

$$AP = \sqrt{x^2 + y^2},$$

and

$$BP = \sqrt{(x-3)^2 + y^2}.$$

Since $AP : BP = 2$, we have

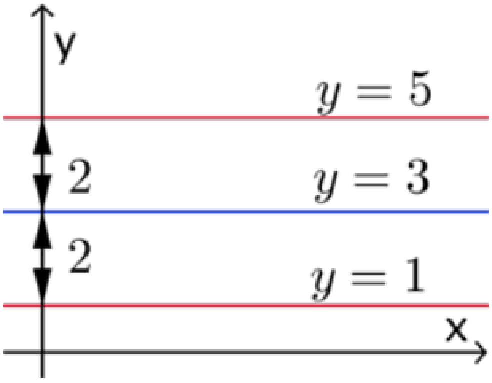
$$\frac{AP}{BP} = 2$$
$$\frac{\sqrt{x^2 + y^2}}{\sqrt{(x-3)^2 + y^2}} = 2$$
$$\frac{x^2 + y^2}{x^2 - 6x + 9 + y^2} = 4$$
$$x^2 + y^2 = 4(x^2 - 6x + 9 + y^2)$$
$$x^2 + y^2 = 4x^2 - 24x + 36 + 4y^2$$
$$3x^2 - 24x + 36 + 3y^2 = 0$$
$$x^2 - 8x + y^2 = -12$$
$$(x^2 - 8x + 16) - 16 + y^2 = -12$$
$$(x-4)^2 + y^2 = 4$$

This is a circle of radius 2 and centre $(4, 0)$.

13 The distance from the point $P(x, y)$ to the line $y = 3$ is 2. Therefore,

$$|y - 3| = 2$$
$$y - 3 = \pm 2$$
$$y = 3 \pm 2$$
$$y = 1 \text{ or } y = 5$$

This pair of lines are shown in red on the diagram below.



14 To solve this problem, draw two circles whose radii are equal to the length of the pipe, and whose centres are the endpoints of the pipe. The pipe can then be moved in a minimum of 3 moves. These are indicated on the diagram below.